

Math 10B with Professor Stankova

Quiz 10; Tuesday, 4/9/2019

Section #203; Time: 11 AM

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Name: \_\_\_\_\_

Circle True or False or leave blank. (1 point for correct answer,  $-1$  for incorrect answer, 0 if left blank)

1. True **FALSE** Shifting the graph of a PDF to the left or right changes the standard deviation.

**Solution:** The standard deviation and variance do not change by shifting the graph.

2. True **FALSE** Chebyshev's inequality can help us when  $0 < k < 1$ .

**Solution:** When  $k < 1$ , we have that  $1/k^2 > 1$  and hence  $1 - 1/k^2 < 0$ , so it doesn't help us.

Show your work and justify your answers. Please circle or box your final answer.

3. (10 points) (a) (7 points) Calculate the standard deviation of  $f(x) = \begin{cases} 3x^{-4} & x \leq -1 \\ 0 & x > -1 \end{cases}$   
(do not use any formulas).

**Solution:** First we need to calculate the mean. The mean is

$$\int_{-\infty}^{\infty} xf(x)dx = \int_{-\infty}^{-1} x(3x^{-4})dx = \frac{-3x^{-2}}{2} \Big|_{-\infty}^{-1} = \frac{-3}{2}.$$

Then the variance is

$$\begin{aligned} \sigma^2 &= \int_{-\infty}^{\infty} x^2 f(x)dx - \frac{(-3)^2}{2^2} = \int_{-\infty}^{-1} 3x^{-2} - \frac{9}{4} \\ &= \frac{-3}{x} \Big|_{-\infty}^{-1} - \frac{9}{4} = 3 - \frac{9}{4} = \frac{3}{4}. \end{aligned}$$

So the standard deviation is  $\frac{\sqrt{3}}{2}$ .

- (b) (3 points) Let  $f$  be a PDF with mean 0 and standard deviation 1. For what value of  $a$  can we say that  $P(-a \leq X \leq a) \geq \frac{8}{9}$ ?

**Solution:** By Chebyshev's inequality, we know that  $P(-a \leq X \leq a) = P(\mu - a\sigma \leq X \leq \mu + a\sigma) \geq 1 - \frac{1}{a^2}$ . So we need that  $\frac{8}{9} = 1 - \frac{1}{a^2}$  so  $a^2 = 9$  and  $a = 3$ .